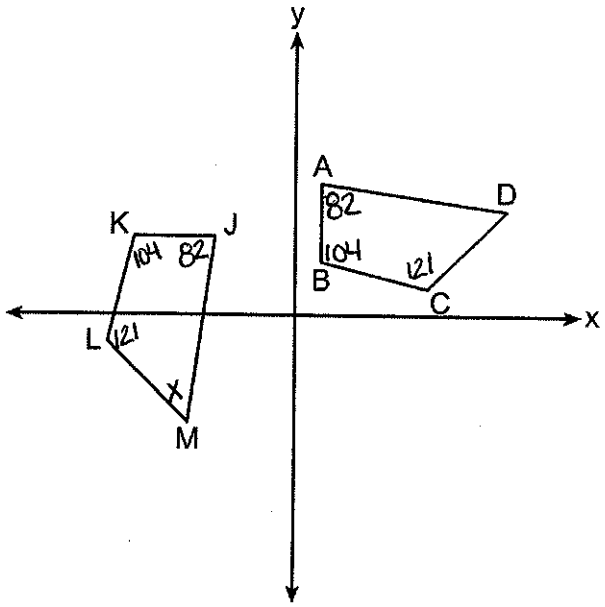
Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 In the diagram below, a sequence of rigid motions maps *ABCD* onto *JKLM*.



$X+121+104+82=360$

$X+307=360$

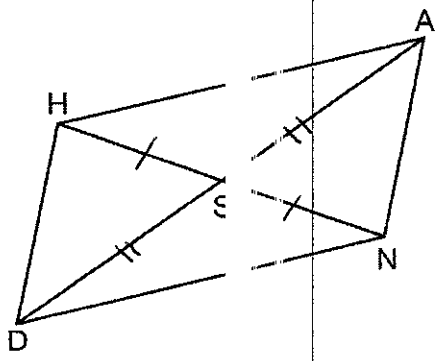
$X=53$

If $m\angle A = 82°$, $m\angle B = 104°$, and $m\angle L = 121°$, the measure of $\angle M$ is

(1) 53°
(3) 104°
(2) 82°
(4) 121°

Use this space for computations.

2 Parallelogram *HAND* is drawn l low with diagonals $\overline{HN}$ and $\overline{AD}$ intersecting at *S*.



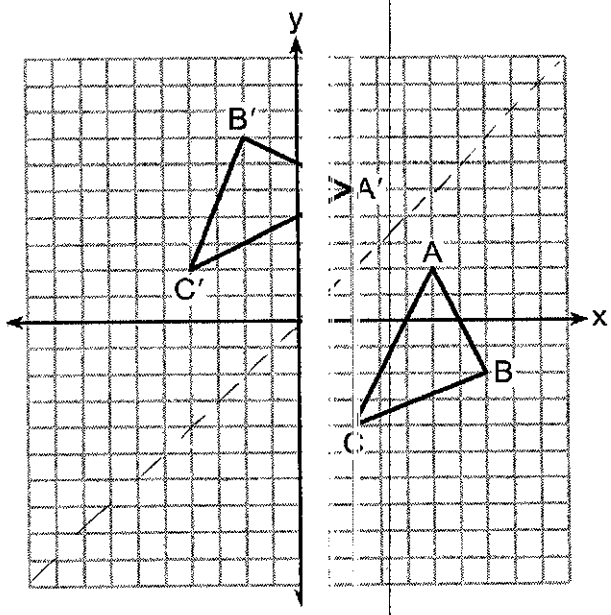
*Diagonals bisect each other

Which statement is always true?

(1) $HN = \frac{1}{2}AD$ (3) $\angle AHS \cong \angle ANS$

(2) $AS = \frac{1}{2}AD$ (4) $\angle HDS \cong \angle NDS$

3 The graph below shows two cong uent triangles, ABC and $A'B'C'$.



Which rigid motion would map ABC onto $\triangle A'B'C'$?

(1) a rotation of 90 degrees cou erclockwise about the origin

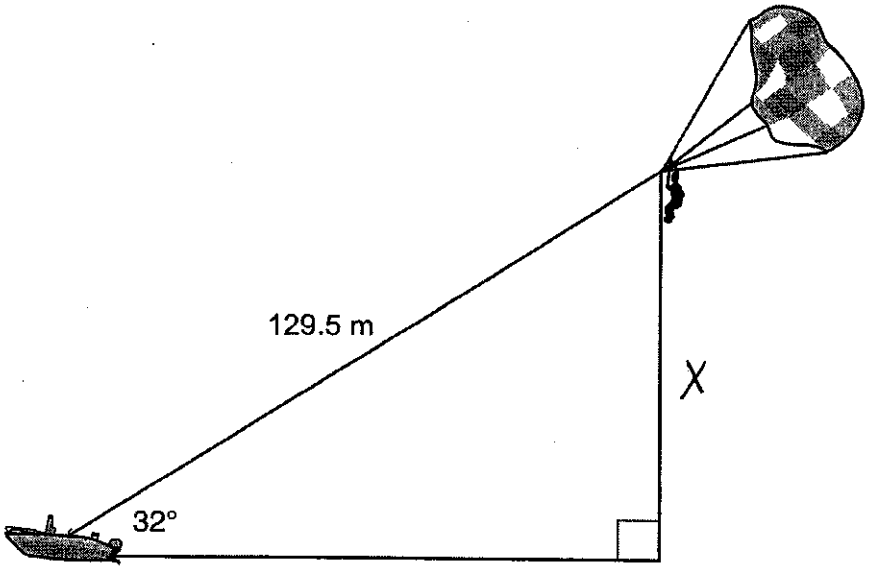
(2) a translation of three units t the left and three units up (only maps A→A')

(3) a rotation of 180 degrees ab it the origin

(4) a reflection over the line $y =$ x

Use this space for computations.

4 A man was parasailing above a lake at an angle of elevation of 32° from a boat, as modeled in the diagram below.



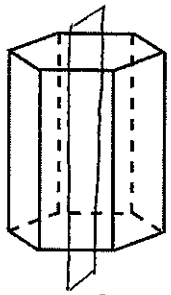
$\frac{\sin 32}{1} = \frac{X}{129.5}$

$X = 68.6$

If 129.5 meters of cable connected the boat to the parasail, approximately how many meters above the lake was the man?

● 68.6
(2) 80.9
(3) 109.8
(4) 244.4

5 A right hexagonal prism is shown below. A two-dimensional cross section that is perpendicular to the base is taken from the prism.

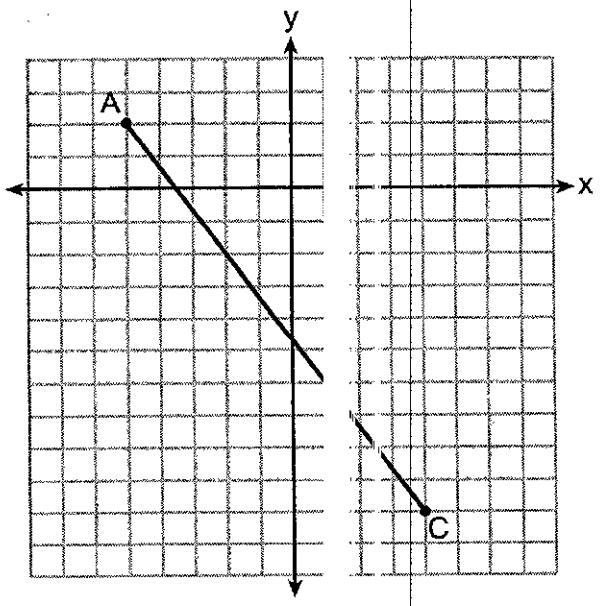


Which figure describes the two-dimensional cross section?

(1) triangle
● rectangle
(3) pentagon
(4) hexagon

Use this space for computations.

6 In the diagram below, $\overline{AC}$ has en points with coordinates $A(-5,2)$ and $C(4,-10)$.



$A(-5,2) \xrightarrow{T\langle 9,-12\rangle} C(4,-10)$

$9\left(\frac{1}{3}\right) \quad -12\left(\frac{1}{3}\right)$

$T\langle 3,-4\rangle$

$A(-5,2) \longrightarrow B(-2,-2)$

If B is a point on $\overline{AC}$ and $AB:BC = 1:2$, what are the coordinates of B?

(1) $(-2,-2)$ (3) $\left(0,-\frac{14}{3}\right)$

(2) $\left(-\frac{1}{2},-4\right)$ (4) $(1,-6)$

7 An ice cream waffle cone can e modeled by a right circular cone with a base diameter of 6 centimeters and a volume of 54.45π cubic centimeters. What is the number of centimeters in the height of the waffle cone?

$V = \frac{1}{3}\pi r^2 h$

$54.45\pi = \frac{1}{3}\pi(3.3)^2 h$

$\frac{54.45\pi}{3.63\pi} = \frac{3.63\pi h}{3.63\pi}$

$15 = h$

(1) $3\frac{3}{4}$ (3) 15

(2) 5 (4) $24\frac{3}{4}$

8 The vertices of $\triangle PQR$ have coordinates $P(2,3)$, $Q(3,8)$, and $R(7,3)$. Under which transformation of $\triangle PQR$ are distance and angle measure preserved?

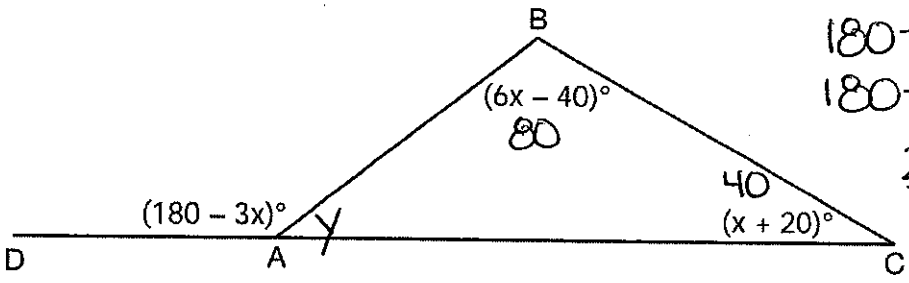
(1) $(x,y) \rightarrow (2x, 3y)$ (3) $(x,y) \rightarrow (2x, y+3)$

(2) $(x,y) \rightarrow (x+2, 3y)$ (4) $(x,y) \rightarrow (x+2, y+3)$

*No dilations just translations

Use this space for computations.

9 In $\triangle ABC$ shown below, side $\overline{AC}$ is extended to point D with $m\angle DAB = (180 - 3x)°$, $m\angle B = (6x - 40)°$, and $m\angle C = (x + 20)°$.



$180-3x=6x-40+x+20$

$180-3x=7x-20$

$\frac{200}{10}=\frac{10x}{10}$

$20=x$

What is $m\angle BAC$?

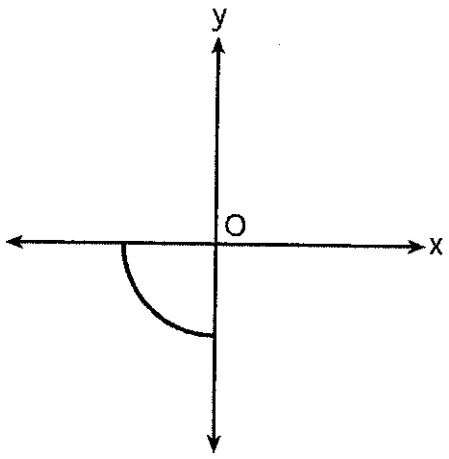
(1) 20°
(2) 40°
(3) 60°
(4) 80°

$y+80+40=180$

$y+120=180$

$y=60$

10 Circle O is centered at the origin. In the diagram below, a quarter of circle O is graphed.



Which three-dimensional figure is generated when the quarter circle is continuously rotated about the y-axis?

(1) cone
(2) sphere
(3) cylinder
(4) hemisphere

Use this space for computations.

11 Rectangle $A'B'C'D'$ is the image of rectangle $ABCD$ after a dilation centered at point A by a scale factor of $\frac{2}{3}$. Which statement is correct?

6 □ 9 4 □ 6

$A=54$ $A=24$

$P=30$ $P=20$

(1) Rectangle $A'B'C'D'$ has a perimeter that is $\frac{2}{3}$ the perimeter of rectangle $ABCD$.

(2) Rectangle $A'B'C'D'$ has a perimeter that is $\frac{3}{2}$ the perimeter of rectangle $ABCD$.

(3) Rectangle $A'B'C'D'$ has an area that is $\frac{2}{3}$ the area of rectangle $ABCD$.

(4) Rectangle $A'B'C'D'$ has an area that is $\frac{3}{2}$ the area of rectangle $ABCD$.

$\left(\frac{2}{2}\right)^2=(1)^2=1$

$\left(\frac{B}{2}\right)^2=\left(\frac{-6}{2}\right)^2=(-3)^2=9$

12 The equation of a circle is $x^2 + y^2 - 6x + 2y = 6$. What are the coordinates of the center and the length of the radius of the circle?

$x^2-6x+9+y^2+2y+1=6+9+1$

$(x-3)^2+(y+1)^2=16$

$C=(3,-1)$ $\sqrt{r^2}=\sqrt{16}$

$r=4$

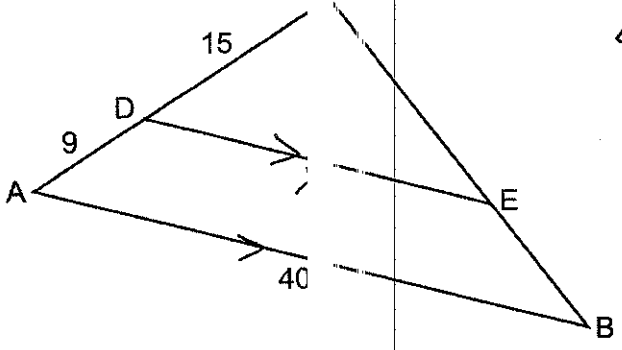
(1) center $(-3,1)$ and radius 4

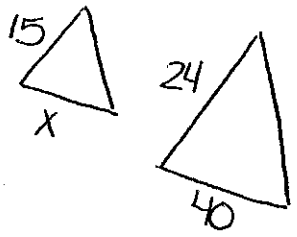
(2) center $(3,-1)$ and radius 4

(3) center $(-3,1)$ and radius 16

(4) center $(3,-1)$ and radius 16

13 In the diagram of $\triangle ABC$ below, $\overline{DE}$ is parallel to $\overline{AB}$, $CD = 15$, $AD = 9$, and $AB = 40$.





$\frac{15}{x}=\frac{24}{40}$

$24x=600$

$x=25$

The length of $\overline{DE}$ is

(1) 15

(2) 24

(3) 25

(4) 30

14 The line whose equation is $3x - 5y = 4$ is dilated by a scale factor of $\frac{5}{3}$ centered at the origin. Which statement is correct?

(1) The image of the line has the same slope as the pre-image but a different y-intercept.

(2) The image of the line has the same y-intercept as the pre-image but a different slope.

(3) The image of the line has the same slope and the same y-intercept as the pre-image.

(4) The image of the line has a different slope and a different y-intercept from the pre-image.

Use this space for computations.

$3x-5y=4$

$\frac{-5y}{-5}=\frac{-3x+4}{-5}$

$y=\frac{3}{5}x-\frac{4}{5}$

pt (0,0) not on line

$m=\frac{3}{5}$ b=different

15 Which transformation would *not* carry a square onto itself?

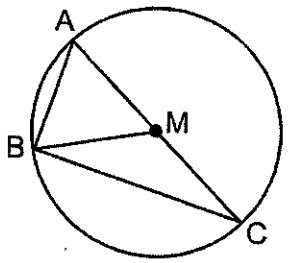
(1) a reflection over one of its diagonals

(2) a 90° rotation clockwise about its center

(3) a 180° rotation about one of its vertices

(4) a reflection over the perpendicular bisector of one side

16 In circle M below, diameter $\overline{AC}$, chords $\overline{AB}$ and $\overline{BC}$, and radius $\overline{MB}$ are drawn.



Which statement is *not* true?

(1) $\triangle ABC$ is a right triangle.

(2) $\triangle ABM$ is isosceles.

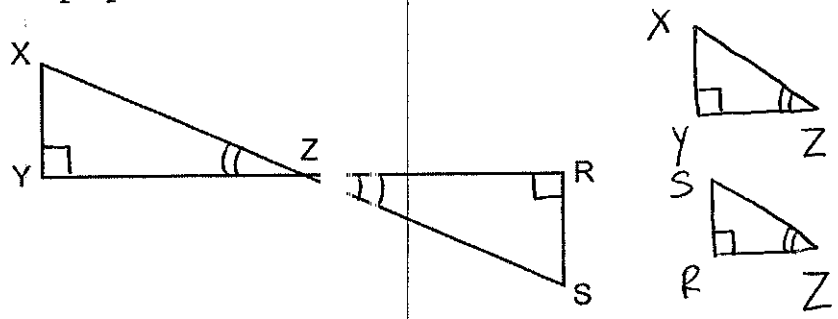
(3) $m\overset{\frown}{BC} = m\angle BMC$

(4) $m\overset{\frown}{AB} = \frac{1}{2}m\angle ACB$

inscribed $\angle = \frac{arc}{2}$

Use this space for computations.

17 In the diagram below, $\overline{XS}$ and $\overline{YR}$ intersect at Z. Segments XY and RS are drawn perpendicular to $\overline{YR}$ to form triangles XYZ and SRZ.



Which statement is always true?

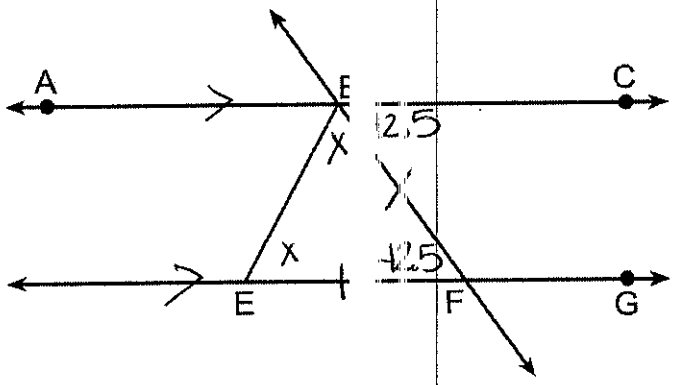
(1) $(XY)(SR) = (XZ)(RZ)$

(2) $\triangle XYZ \cong \triangle SRZ$

(3) $\overline{XS} \cong \overline{YR}$

(4) $\frac{XY}{SR} = \frac{YZ}{RZ}$

18 As shown in the diagram below, $\overleftrightarrow{BC} \parallel \overleftrightarrow{EFG}$ and $\overline{BF} \cong \overline{EF}$.



$X + X + 42.5 = 180$

$-42.5 \quad -42.5$

$\frac{2X}{2} = \frac{137.5}{2}$

$X = 68.75$

If $m\angle CBF = 42.5°$, then $m\angle EF$ is

(1) 42.5°

(2) 68.75°

(3) 95°

(4) 137.5°

19 A parallelogram must be a rhombus if its diagonals

(1) are congruent

(2) bisect each other

(3) do not bisect its angles

(4) are perpendicular to each other

Use this space for computations.

20 What is an equation of a line which passes through (6,9) and is perpendicular to the line whose equation is $4x - 6y = 15$?

● $y - 9 = -\frac{3}{2}(x - 6)$

(2) $y - 9 = \frac{2}{3}(x - 6)$

(3) $y + 9 = -\frac{3}{2}(x + 6)$

(4) $y + 9 = \frac{2}{3}(x + 6)$

$4x - 6y = 15$

$\frac{-6y}{-6} = \frac{-4x + 15}{-6}$

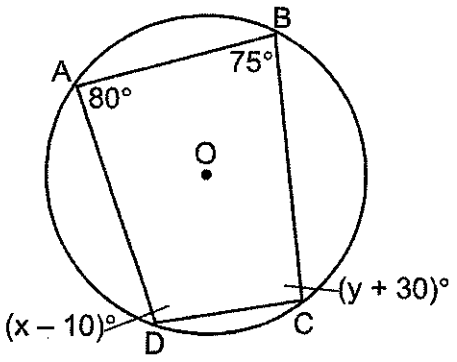
$y = \frac{2}{3}x - \frac{15}{6}$

$y - y = m(x - x)$

$y - 9 = -\frac{3}{2}(x - 6)$

21 Quadrilateral $ABCD$ is inscribed in circle O, as shown below.

$m = \frac{2}{3}$ $\perp m = -\frac{3}{2}$



$80 + y + 30 = 180$

$y + 110 = 180$

$y = 70$

If $m\angle A = 80°$, $m\angle B = 75°$, $m\angle C = (y + 30)°$, and $m\angle D = (x - 10)°$, which statement is true?

(1) $x = 85$ and $y = 50$

(2) $x = 90$ and $y = 45$

(3) $x = 110$ and $y = 75$

● $x = 115$ and $y = 70$

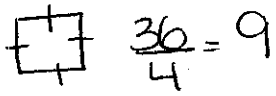
22 A regular pyramid has a square base. The perimeter of the base is 36 inches and the height of the pyramid is 15 inches. What is the volume of the pyramid in cubic inches?

(1) 180

● 405

(3) 540

(4) 1215



$\frac{36}{4} = 9$

$V = \frac{1}{3}Bh$

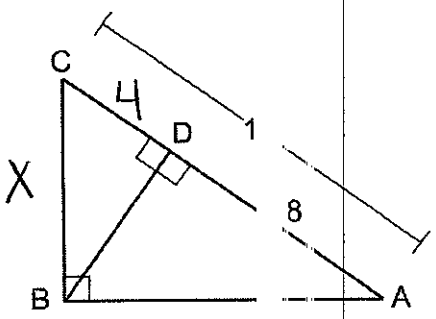
$\frac{1}{3}lwh$

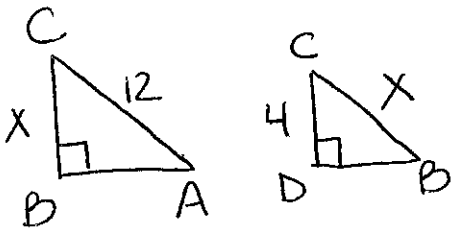
$\frac{1}{3}(9)(9)(15)$

405

Use this space for computations.

23 In the diagram below of $\triangle ABC$, ABC is a right angle, $AC = 12$, $AD = 8$, and altitude $\overline{BD}$ is drawn.





$\frac{X}{12} = \frac{4}{X}$

$\sqrt{X^2} = \sqrt{48}$

$\sqrt{16}\sqrt{3}$

$4\sqrt{3}$

What is the length of $\overline{BC}$?

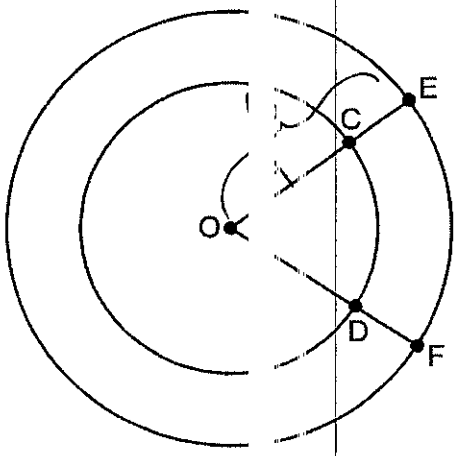
(1) $4\sqrt{2}$

(3) $4\sqrt{5}$

● $4\sqrt{3}$

(4) $4\sqrt{6}$

24 In the diagram below, two concentric circles with center O, and radii $\overline{OC}$, $\overline{OD}$, $\overline{OCE}$, and $\overline{ODF}$ are drawn.



$\frac{\text{image } EO}{\text{preimage } CO} = \frac{6}{4} = 1.5$

If $OC = 4$ and $OE = 6$, which relationship between the length of arc EF and the length of arc CD is always true?

(1) The length of arc EF is 2 units longer than the length of arc CD.

(2) The length of arc EF is 4 units longer than the length of arc CD.

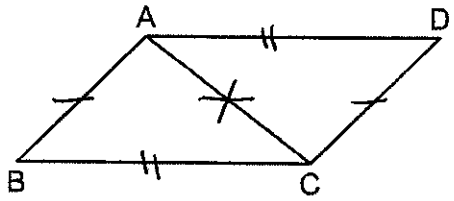
● The length of arc EF is 1.5 times the length of arc CD.

(4) The length of arc EF is 2.0 times the length of arc CD.

Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

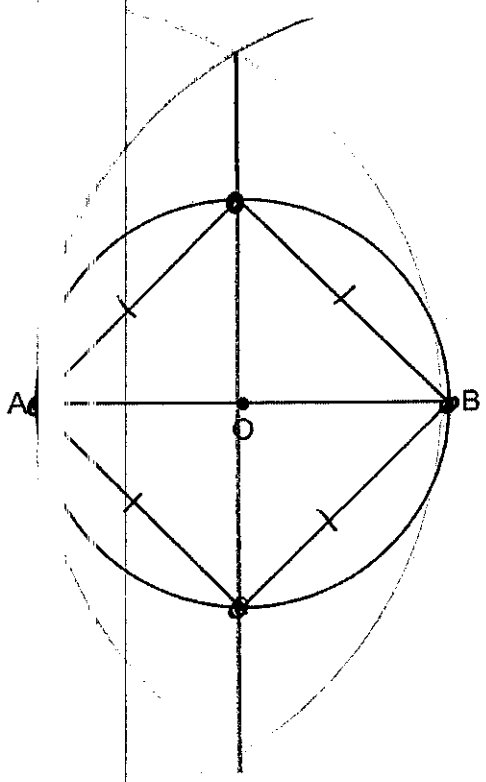
25 Given: Parallelogram $ABCD$ with diagonal $\overline{AC}$ drawn



Prove: $\triangle ABC \cong \triangle CDA$

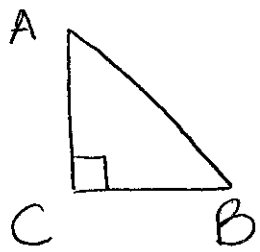
S	R
① Ilogram ABCD	① Given
② $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$	② In a Ilogram opposite sides are $\cong$
③ $\overline{AC} \cong \overline{AC}$	③ Reflexive
④ $\triangle ABC \cong \triangle CDA$	④ SSS $\cong$ SSS

26 The diagram below shows cir e O with diameter $\overline{AB}$. Using a compass and straightedge, construct a square that is inscri d in circle O. [Leave all construction marks.]



27 Given: Right triangle ABC with right angle at C

If sin A increases, does cos B increase or decrease? Explain why.



ex. sin 30 = cos 60

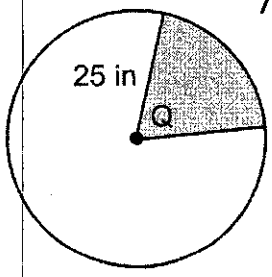
sin 50 = cos 40

sine and cosine are cofunctions

*The ∠s are complementary

therefore when sin A increases, cos B increases

28 In the diagram below, the circle has a radius of 25 inches. The area of the *unshaded* sector is 500π in^2.



$A.S. = \frac{n}{360} \cdot \pi r^2$

$\frac{500\pi}{1} = \frac{n\pi(25)^2}{360}$

$\frac{180000\pi}{625\pi} = \frac{625\pi n}{625\pi}$

$288 = n$

Determine and state the degree measure of angle Q, the central angle of the shaded sector.

360
− 288
72

29 A machinist creates a solid steel part for a wind turbine engine. The part has a volume of 1015 cubic centimeters. Steel can be purchased for \$0.29 per kilogram, and has a density of $7.95 g/cm^3$.

If the machinist makes 500 of these parts, what is the cost of the steel, to the *nearest dollar*?

$D = \frac{M}{V}$

$1000\ g = 1\ kg$

$\frac{7.95}{1} = \frac{M}{1015}$

$M = \frac{8069.25}{1000}\ g = 8.06925\ kg$

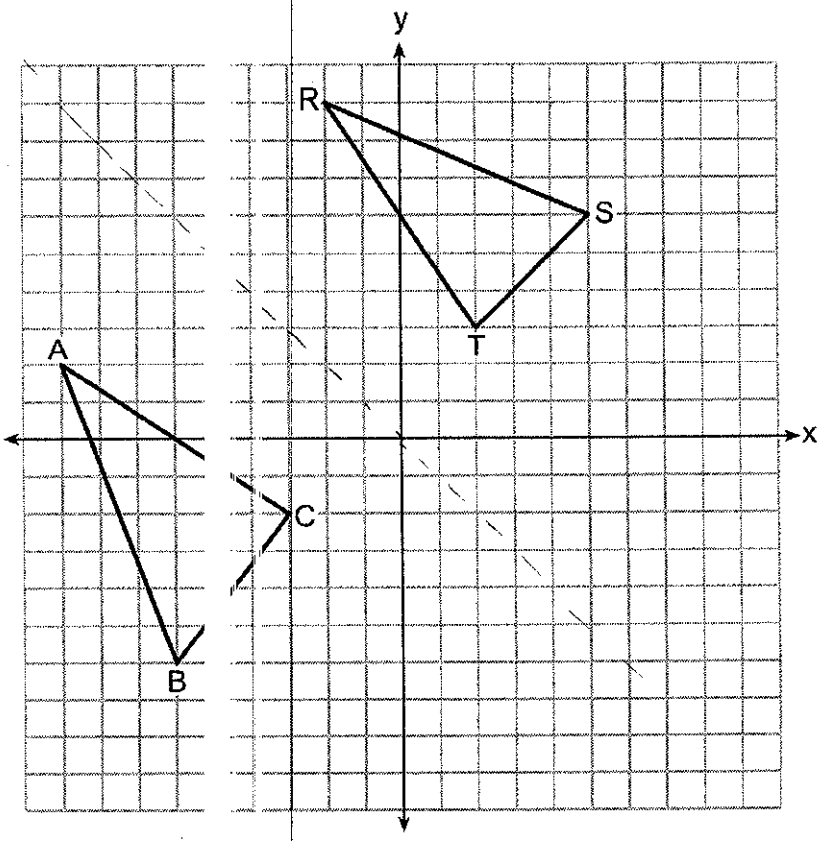
8.06925×500

4034.625

$4034.625 \times .29$

\$1170

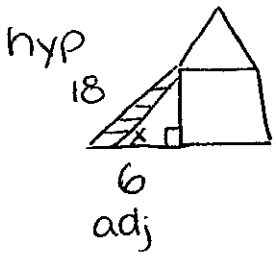
30 In the graph below, $\triangle ABC$ has coordinates $A(-9,2)$, $B(-6,-6)$, and $C(-3,-2)$, and $\triangle RST$ has coordinates $R(-2,9)$, $S(5,6)$, and $T(2,3)$.



Is $\triangle ABC$ congruent to $\triangle RST$? Use the properties of rigid motions to explain your reasoning.

No, there is no specific rigid motion that would map $\triangle ABC$ onto $\triangle RST$. A rigid motion preserves distance and angle measure.

31 Bob places an 18-foot ladder 6 feet from the base of his house and leans it up against the side of his house. Find, to the *nearest degree*, the measure of the angle the bottom of the ladder makes with the ground.



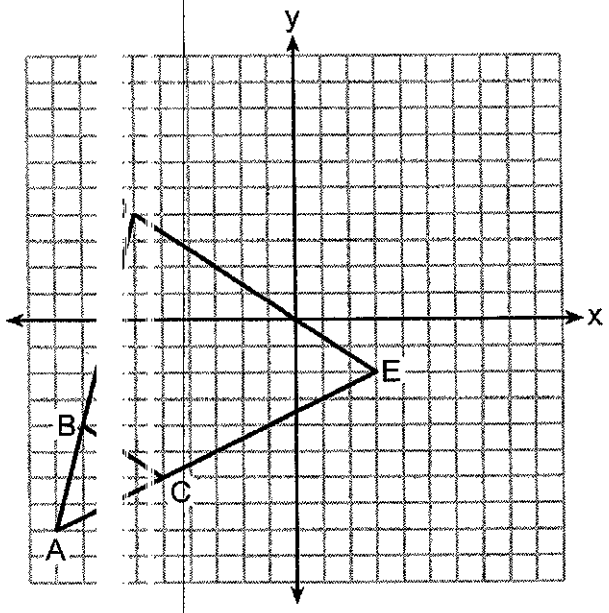
$\cos x = \frac{6}{18}$

$x = 71$

Part III

Answer all 3 questions in th part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, incl ding appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the informati n provided for each question to determine your answer. Note that diagrams are not neces rily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs nd drawings, which should be done in pencil. [12]

32 Triangle *ABC* and triangle *ADE* re graphed on the set of axes below.



Describe a transformation that aps triangle *ABC* onto triangle *ADE*.

Dilation of 3 centered at point A

Explain why this transformatio makes triangle *ADE* similar to triangle *ABC*.

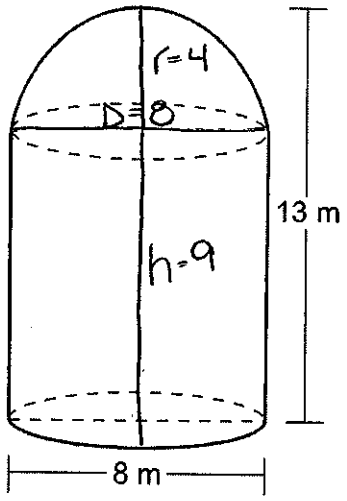
A dilation cha ges the size but preserves angle measu e

$\angle A \cong \angle A$ reflexive AA ~

$\angle ABC \cong \angle ADE$

$\angle ACB \cong \angle AED$

33 A storage tank is in the shape of a cylinder with a hemisphere on the top. The highest point on the inside of the storage tank is 13 meters above the floor of the storage tank, and the diameter inside the cylinder is 8 meters. Determine and state, to the *nearest cubic meter*, the total volume inside the storage tank.



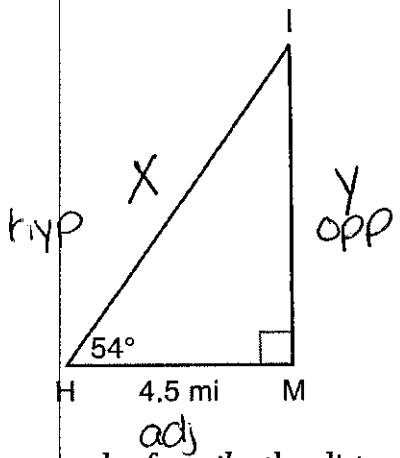
$V_{hemisphere} + V_{cylinder}$

$\frac{1}{2}\left(\frac{4}{3}\pi r^3\right) + \pi r^2 h$

$\frac{1}{2}\left(\frac{4}{3}\pi\right)(4)^3 + \pi(4)^2(9)$

586

34 As shown in the diagram below, an island (I) is due north of a marina (M). A boat house (H) is 4.5 miles due west of the marina. From the boat house, the island is located at an angle of 54° from the marina.



Determine and state, to the *nearest tenth of a mile*, the distance from the boat house (H) to the island (I).

$\frac{\cos 54}{1} = \frac{4.5}{X}$

$X \cos 54 = 4.5$

$\frac{X \cos 54}{\cos 54} = \frac{4.5}{\cos 54}$

$\boxed{X = 7.7}$

Determine and state, to the *nearest tenth of a mile*, the distance from the island (I) to the marina (M).

$\frac{\tan 54}{1} = \frac{Y}{4.5}$

$\boxed{Y = 6.2}$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

35 In the coordinate plane, the vertices of triangle *PAT* are $P(-1,-6)$, $A(-4,5)$, and $T(5,-2)$. Prove that $\triangle PAT$ is an isosceles triangle. [The use of the set of axes on the next page is optional.]

$PA = \sqrt{(-1-4)^2+(-6-5)^2}$

$\sqrt{(3)^2+(-11)^2}$

$\sqrt{9+121}$

$\sqrt{130}$

$AT = \sqrt{(-4-5)^2+(5-2)^2}$

$\sqrt{(-9)^2+(7)^2}$

$\sqrt{81+49}$

$\sqrt{130}$

$TP = \sqrt{(-1-5)^2+(-6-2)^2}$

$\sqrt{(-6)^2+(-4)^2}$

$\sqrt{36+16}$

$\sqrt{52}$

$\triangle PAT$ is isosceles b/c there are 2 $\cong$ sides

State the coordinates of *R* so that quadrilateral *PART* is a parallelogram. $R(2,9)$

Question 35 is continued on the next page.

Question 35 continued

Prove that quadrilateral *PART* is a parallelogram.

$PA = \frac{-11}{3}$ $AR = \frac{11}{6} = \frac{2}{3}$

$RT = \frac{-11}{3}$ $TP = \frac{11}{6} = \frac{2}{3}$

Quad PART is a llogram b/c opposite sides are ||